The Evolution and Efficiency of Oblique MHD Cosmic-Ray Shocks: Two-Fluid Simulations

Adam Frank[1], T. W. Jones[1] and Dongsu Ryu[2]

[1]Department. of Astronomy, University of Minnesota, Minneapolis, MN 55455
[2]Department of Astronomy & Space Science, Chungnam National University, Daejeon 305-764, Korea

Abstract

Using a new, second-order accurate numerical method we present dynamical simulations of oblique MHD cosmic ray (CR) modified plane shock evolution using the two-fluid model for diffusive particle acceleration. The numerical shocks evolve to published analytical steady state properties. In order to probe the dynamical role of magnetic fields we have explored for these time asymptotic states the parameter space of upstream fast mode Mach number, M_f, and plasma β, compiling the results into maps of dynamical steady state CR acceleration efficiency, ϵ_c. These maps, along with additional numerical experiments, show that ϵ_c is reduced through the action of compressive work on tangential magnetic fields in CR-MHD shocks. Thus ϵ_c in low β, moderate M_f shocks tends to be smaller in quasi perpendicular shocks than it would be high β shocks of the same M_f. This result supports earlier conclusions that strong, oblique magnetic fields inhibit diffusive shock acceleration. For quasi parallel shocks with $\beta < 1$, on the other hand, ϵ_c seems to be increased at a given M_f when compared to high β shocks. The apparent contradiction to the first conclusion results, however, from the fact that for small β quasi parallel shocks, the fast mode Mach number is not a good measure of compression through the shock. That is better reflected in the sonic Mach number, which is greater. Acceleration efficiencies for high and low β having comparable sonic Mach numbers are more similar.

Time evolution of CR-MHD shocks is qualitatively similar to CR-gasdynamical shocks. However, several potentially interesting differences are apparent. We have run simulations using constant, and non-isotropic, obliquity dependent forms of the diffusion coefficient κ. Comparison of the results show that while the final steady states achieved are the same in each case, the history of CR-MHD shocks can be strongly modified by variations in κ and, therefore, in the acceleration timescale. Also, the coupling of CR and MHD in low β, oblique shocks substantially influences the transient density spike that forms in strongly CR-modified shocks. We find that inside the density spike a MHD slow mode wave is generated that eventually steepens into a shock. A strong shear layer develops within the density spike, driven by MHD stresses. We conjecture that currents in the shear layer could, in non-planar flows, result in enhanced particle acceleration through drift acceleration.

1. **Introduction**

In the past several years nonlinear theories of diffusive shock acceleration have demonstrated the potential importance of cosmic-ray (CR) dynamical feedback on the evolution of shock structures (see reviews by Blandford & Eichler 1987; Berezhko & Krymskii 1988; Jones & Ellison 1991). A range of methods has been successfully applied to show that CR pressures can become sufficient to modify shock structures substantially (*e.g.,* Drury & Völk 1981; Achterberg *et al.* 1984; Ellison & Eichler 1984; Webb, Drury, and Völk 1986; Falle & Giddings 1987; Kang & Jones 1990, 1991). In some cases the modifications can even produce smooth CR dominated shocks with no entropy generating gas sub-shock. Numerical simulations using a variety of techniques have also demonstrated the importance of time-dependent effects in the determination of CR modified shock properties (e.g., Drury & Falle 1987; Falle & Giddings 1989; Kang & Jones 1991; Duffy 1992; Kang, Jones, & Ryu, 1992; Ryu, Kang & Jones 1993).

Through all of this the presence of a large-scale magnetic field was generally understood to be an essential physical ingredient in CR shocks, since magnetic field fluctuations (small-scale Alfvén waves, or "Alfvénic turbulence" to avoid confusion with large scale waves) mediate the CR propagation. But the magnetic fields are difficult to treat explicitly and fully, especially when those fields play a direct dynamical role. So, most studies of CR modified shocks have focused on pure gasdynamical models of the flows. That may be adequate if the fields are sufficiently weak, and/or the large-scale field is aligned parallel to the shock normal, so that sonic-mode shock dynamics applies. Except for modeling the spatial dependence of the CR diffusion coefficient, explicit treatment of the magnetic field may not always be generally necessary when the shock speed in the preshock (upstream) fluid, u_s, and the preshock sound speed, a are large compared to the preshock Alfvén speed, b. The second of these conditions is commonly expressed as $\beta = a^2/b^2 \gg 1$. But, this condition is likely to be violated, perhaps strongly, in many environments where particle acceleration occurs (*e.g.,* Slavin & Cox 1992). Even if the magnetic field and shock normal are parallel, treatment of CR shocks in flows with $\beta \lesssim 1$ may depend in significant ways on β, especially if the Alfvénic Mach number is not very large. One must be concerned, for example, with the finite momentum and energy carried by the CR-mediating Alfvénic turbulence (*e.g.,* Völk, Drury & McKenzie 1984; Jones 1993a)

Postponing for now discussion of the more complicated topics in general magnetohydrodynamical (MHD) flows, our purpose in the present paper is to examine some straightforward dynamical influences of oblique magnetic fields on plane CR shocks. There are several obvious ways in which oblique fields may be important. If the pressure in the tangential field is comparable to the gas pressure of the incoming flow, that changes the jump conditions to be satisfied through the shock and must, therefore, influence the transfer of energy and momentum to CR within the shock (*e.g.,* Webb 1983). The anisotropic nature of MHD stresses in oblique shocks leads to the existence of fast, intermediate and slow wave mode families, each with distinct behaviors. That can complicate the shock properties and lead to the generation of distinctly MHD time dependent structures in a

shock evolving in response to CR acceleration. This increase in the degrees of freedom at the shock also augments the modes through which the shock structure may become dynamically unstable (*e.g.,* Chalov 1988; Zank *et al.* 1990). The CR diffusion coefficient may be anisotropic relative to the field, affecting the rates at which CR are accelerated. If the diffusion is described by standard kinetic theory models, the acceleration rate at oblique shocks could be much faster than at otherwise comparable parallel shocks (Jokipii 1987; Ostrowski 1988), although the resulting time-asymptotic test-particle form for the particle spectrum is apparently unchanged (Bell 1978), at least for nonrelativistic shocks (Kirk & Heavens 1989). This change in the acceleration rate should influence the dynamical properties of the shock at intermediate times, before a dynamical equilibrium is achieved, even if the field is weak and not directly involved dynamically. On the other hand, Baring, Ellison & Jones (1993) have shown that the efficiency of thermal particle injection into the CR population can be decreased by perpendicular magnetic field components. From these examples it is clear that as a step to understand the fundamental nature of diffusive shock acceleration in more realistic astrophysical settings, full time dependent MHD calculations are needed.

Two-fluid models of CR transport along the lines developed by Drury & Völk (1981) and others are an efficient means to begin such explorations. These models treat transport of the CR through an energy conservation equation derived from the kinetic, diffusion-advection equation (*e.g.,* Skilling 1975). The CR momentum distribution function itself is not followed in detail, except possibly through *a priori* models (*e.g.,* Jones & Kang 1993; Duffy *et al.* 1993). Two-fluid methods are meant to be used primarily for dynamical studies. They enable one to calculate economically the dynamical features of flows within the constraints imposed by the need to estimate *a priori* some closure parameters for the CR (see Drury 1983; Kang & Jones 1991; Kang 1993). In particular one must model the adiabatic index for the CR and the energy-weighted mean diffusion coefficient. To the extent that the solutions depend on those parameters (see, *e.g.,* Achterberg *et al.* 1984; Kang & Jones 1990) in ways that are not modelled, two-fluid models should not be used for quantitative prediction. On the other hand, the dependence of two-fluid solutions on these parameters for gasdynamic CR problems has been reasonably well studied (*e.g.,* Achterberg *et al.* 1984; Kang & Jones 1990) and the basic validity of two-fluid dynamical solutions compared to more complete, diffusion-advection solutions for the CR momentum distribution demonstrated (*e.g.,* Falle & Giddings 1987; Kang & Jones 1991; Ryu, Kang & Jones 1993). Some recent calculations even suggest that relatively simple models connecting the properties of the particle distribution to the closure parameters might provide adequate and internally self-consistent procedures for eliminating most limitations (Duffy *et al.* 1993). In any case, our purposes here are to explore the dynamical consequences of the magnetic field in these problems. Two-fluid methods would seem to be both adequate and appropriate for that aim.

Webb (1983) first applied two-fluid methods to equilibrium, steady state CR-MHD modified shock structures for the case where the upstream gas is cold (its pressure can be ignored) and the magnetic field is oblique. Webb's analytic calculations demonstrated that, as for the CR modified gasdynamical flows, both sub-shock containing and smoothed, CR dominated solutions to the CR-MHD shock equations were possible. However, his solutions

suggested that shock CR acceleration was less effective when the upstream tangential component of the field was strong. Subsequently Kennel, Edmiston & Hada (1985) and Webb, Drury, and Völk 1986 (hereafter, WDV) calculated steady state two-fluid CR-MHD modified shock structures with finite upstream gas pressures. From their analysis, which included $\beta = 5$ and $\beta = 1$ cases and a range of fast mode Mach numbers, M_f, they concluded that the efficiency of CR acceleration was suppressed when the obliquity of the upstream magnetic field was high. They also found that the reduction in CR acceleration efficiency was most pronounced in the lower β ($\beta = 1$) flows of low fast mode Mach number ($M_f < 6$). For high β ($\beta = 5$) flows the CR acceleration efficiency was only weakly affected by changes in the upstream magnetic field angle regardless of the fast mode Mach number.

As an extension of the previous work, we report in this paper the results of *time dependent* CR-MHD two-fluid simulations using a new numerical code. In an earlier work (Frank, Jones, & Ryu 1993) we presented tests of this code against Webb's cold gas analytical models as well as more general internal checks on our code's ability to evolve oblique shocks to self-consistent steady state CR-MHD structures. The simulations in that paper also confirmed Webb's cold gas solutions, presented preliminary results from more general CR-MHD flows and determined the convergence properties of our numerical method. In the present paper we explore the time evolution of finite gas pressure ($P_g \neq 0$) models of CR-MHD modified oblique shocks and their relation to the WDV steady state calculations. We will also explore in more detail questions raised in WDV concerning the role of both the obliquity of the field and the tangential field strength in determining the CR acceleration efficiency. More general time dependent effects unique to CR-MHD modified shocks will also be examined along with their potential importance for CR studies. As in WDV we will concentrate on those flows with high enough Alfvénic Mach number that switch-on and switch-off shocks do not occur. In the regime where those special shocks form, CR scattering centers (the Alfvénic turbulence) move at speeds comparable to the gas, requiring that the time dependent wave field dynamics be included in the calculations. For the present we restrict ourselves to flows where the scattering centers can be considered moving with the fluid. Jun, Clarke & Norman (1993) have also recently simulated CR-MHD shocks using two-fluid methods. Their results are complementary to ours, since they restricted their analysis to the special case of perpendicular (magnetosonic) shocks, so that they could not explore a number of questions address here.

The remainder of the paper is organized as follows. Section 2 introduces the fundamental equations governing CR modified MHD flows and discusses their application to our methods. An appendix at the end discusses these numerical methods in more detail. Section 3 compares the results of our models with those of WDV and explores issues concerning the dependence of CR shock acceleration on upstream magnetic field properties. In §4 we examine the general time dependent features of CR-MHD modified shocks as seen in our simulations, while we provide in §5 a discussion of these results in light of previous studies and their potential for application in higher dimensional flows. We present our conclusions in §6.

2. **Methods**

We solve the equations of non-relativistic ideal MHD for one dimensional flow in Cartesian coordinates (*e.g.,* Jeffrey 1968). As with gasdynamical models the conservation equations are modified to include momentum and energy source terms from CR feedback (*e.g.,* Drury & Völk, 1981; Jones & Kang, 1990). The MHD equations are written in conservative, vector form as

$$\frac{\partial \boldsymbol{q}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S}, \tag{2.1}$$

with

$$\boldsymbol{q} = \begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \\ \rho u_z \\ B_y \\ B_z \\ E \end{pmatrix}, \tag{2.2}$$

and

$$\boldsymbol{F} = \begin{pmatrix} \rho u_x \\ \rho u_x^2 + P^* - B_x^2 \\ \rho u_x u_y - B_x B_y \\ \rho u_x u_z - B_x B_z \\ B_y u_x - B_x u_y \\ B_z u_x - B_x u_z \\ (E + P^*) u_x - B_x (B_x u_x + B_y u_y + B_z u_z) \end{pmatrix}, \tag{2.3}$$

while the CR source term vector is

$$\boldsymbol{S} = - \begin{pmatrix} 0 \\ \partial P_c / \partial x \\ 0 \\ 0 \\ 0 \\ 0 \\ u_x \partial P_c / \partial x + S_e \end{pmatrix}. \tag{2.4}$$

In two-fluid models the CR are themselves treated as a massless, diffusive fluid through a conservation equation for the CR energy, E_c, derived by taking the second moment of the diffusion-advection equation (*e.g.,* Achterberg 1982); namely,

$$\frac{\partial E_c}{\partial t} + \frac{\partial}{\partial x}\left(u_x E_c - \langle \kappa \rangle \frac{\partial E_c}{\partial x} \right) = - P_c \frac{\partial u_x}{\partial x} + S_e. \tag{2.5}$$

A more general treatment would replace equation [2.5] with the diffusion-advection equation itself. Its moments would be calculated independently. In these relations

$$P^* = P_g + \frac{1}{2}(B_x^2 + B_y^2 + B_z^2) = P_g + \frac{1}{2}B^2, \tag{2.6}$$

$$E = \frac{1}{2}\rho(u_x^2 + u_y^2 + u_z^2) + \frac{1}{(\gamma_g - 1)}P + \frac{1}{2}(B_x^2 + B_y^2 + B_z^2), \tag{2.7}$$

$$P_c = (\gamma_c - 1) E_c, \tag{2.8}$$

and the magnetic field components are expressed in rationalized units

$$B \rightarrow \frac{B}{\sqrt{4\pi}}, \tag{2.9}$$

so that the Alfvén speed, $b = B/\sqrt{\rho}$. In the expressions presented above the following definitions hold: ρ is the mass density; u_x, B_x and u_y, u_z, B_y, B_z are the components of velocity and magnetic field parallel and perpendicular to shock front; P_g, γ_g and P_c, γ_c are the gas and CR pressures and adiabatic indices, and $\langle\kappa\rangle$ is the energy-weighted, CR diffusion coefficient parallel to the shock normal (Drury 1983). (For simplicity, we will henceforth omit the brackets around κ.) It is not necessary in two-fluid models to assume that either of γ_c or κ is a constant or that κ derives from an energy independent diffusion process. As described earlier, they do depend on *a priori* models, however. The quantity S_e is a term that allows direct energy transfer from the gas to the CR, such as through the injection of thermal particles into the CR population (*e.g.,* Jones & Kang 1990). This is introduced for completeness, but for the present, we set $S_e = 0$. We note in passing, however, that in the course of this investigation we carried out analogous calculations with $S_e \neq 0$. Behaviors of these shocks were not qualitatively different from those we will describe (*cf.* Kang & Jones 1990, 1991). In this discussion $\gamma_g = 5/3$, while the CR closure parameter γ_c, which depends upon the mix of non-relativistic ($\gamma_c \rightarrow 5/3$) and relativistic ($\gamma_c \rightarrow 4/3$) particles in the CR population, will be treated as an input parameter. In general both γ_c and κ are properties of the solution, so the need to specify their values *a priori* is the principal restriction of the two-fluid model.

In the interest of simplicity we will not consider here flows with rotations of the shock-normal/magnetic-field plane between the initial upstream and downstream states, although the numerical code is quite capable of handling such features. Thus, without further loss of generality we can place the magnetic field in the X-Z plane, $\vec{B} = (B_x, 0, B_z) = B(cos\theta, 0, sin\theta)$. We will also select a reference frame so that there is no upstream tangential velocity; $u_y = u_z = 0$. All of the simulations discussed here are piston driven shock tubes. We establish the flows by projecting magnetized fluid with embedded CR in from the right boundary, using continuous boundary conditions and reflecting it off a wall (piston) at the left boundary. The tangential magnetic field at the left boundary is "mirrored" in a manner analogous to the gas density. Thus, there is no current sheet on the piston surface. In other words, the simulations can be considered as those of two identical, colliding clouds with magnetic field "continuous" across them. The CR pressure is given continuous boundary conditions on both ends of the grid.

In general, spatial diffusion will differ in directions parallel and perpendicular to the magnetic field. Diffusive acceleration depends on the diffusion along the shock normal, since that determines the rate at which particles re-cross the shock. Thus, the appropriate diffusion coefficient in equation [2.5] will take the form (*e.g.,* Drury 1983; Webb 1983),

$$\kappa = \kappa_{\parallel} \cos^2\theta + \kappa_{\perp} \sin^2\theta, \tag{2.10}$$

where the directions $\parallel$ and $\perp$ refer to the magnetic field direction. Note, of course, that θ is generally a changing function of space and time. In standard kinetic theory scattering

models of particle diffusion the ratio $\kappa_{\|}/\kappa_{\perp} \sim 1 + (\omega\tau)^2$, where ω, and τ are the particle gyro frequency and collision time, respectively (*e.g.,* Jokipii 1987; Zank *et al.* 1990). We will adopt the form for κ in equation [2.10] in our simulations.

Previous studies of CR modified shocks have shown that the time required to evolve towards dynamical equilibrium scales with the so-called diffusion time, t_d, (*e.g.,* Drury & Falle 1986; Jones & Kang 1990), which in the present case is conveniently expressed as

$$t_d = \frac{\kappa}{u_s^2} = \frac{x_d}{u_s}, \tag{2.11}$$

where u_s is the shock speed in the upstream fluid (see equation [3.7]) and x_d is known as the diffusion length. x_d is a measure of the breadth of the CR precursor to the shock. It is important that computational resolution be fine enough to resolve the flow features on this scale. For our discussion we define, therefore, the resolution ratio of each simulation to be

$$n_r = \frac{x_d}{\Delta x}, \tag{2.12}$$

where Δx is the size of a computational zone. Frank, Jones & Ryu (1993) found that provided $n_r > 10 - 20$, our code produced accurate and converged solutions. When $\kappa_{\|} \neq \kappa_{\perp}$, t_d is not well defined, since κ changes through a shock. To be specific, however, we will adopt the convention of using the value of κ from equation [2.10] upstream of the shock to define t_d in our discussions.

Our numerical method solves equation [2.1] through the second order accurate, Eulerian finite difference, "Total Variation Diminishing" (TVD) scheme. The TVD scheme was originally designed for gasdynamics by Harten (1983) and extended to MHD by Brio and Wu (1988). In this paper, we use the version of the MHD-TVD code which was subsequently improved and fully tested by Ryu & Jones (1993). The pure MHD ($\boldsymbol{S} = 0$) form of the equation is solved with the aid of an approximate MHD Riemann solver used to estimate the flux, $\boldsymbol{F}$. CR source corrections, $\boldsymbol{S}$, are added separately in a manner preserving second order accuracy. Shocks and other discontinuities are generally sharply resolved within a few zones. The CR energy equation is solved separately using a second order combined Lagrangian Crank-Nicholson and monotone remap scheme. This mixed scheme is efficient and provided us much better stability and accuracy than a straight Eulerian Crank-Nicholson approach. In the appendix we present the numerical method in more detail. A pure gasdynamical version of the code was tested extensively against both analytical steady state solutions and numerical time dependent models calculated with our well-tested PPM two-fluid code (Jones & Kang 1990), all with excellent agreement. The pure MHD code was tested using a nonlinear MHD Riemann solver against a variety of planar shock tube problems involving all three wave mode families (for details, see Ryu & Jones 1993).

3. **Results**

We will now summarize results from these simulations, beginning with some properties of flows that were continued until the state immediately downstream of the shock was no

longer evolving; *i.e.,* a time asymptotic state. Then we will explore behaviors of CR-MHD shocks at intermediate times, while the post shock flow is still developing. There are also some notable features that continue to evolve downstream of the equilibrium shock.

To describe the flows there are four parameters needed in addition to the CR "fluid" properties, γ_c and κ. Thus, we parameterize our upstream state by the following dimensionless quantities,

$$\begin{aligned} \beta &= \frac{a^2}{b^2} = \frac{\gamma_g P_g}{B^2} \\ N &= \frac{P_c}{P_g + P_c}, \\ M_f &= \frac{|u_x|}{c_f}, \\ \theta_o &= \tan^{-1}(\frac{B_z}{B_x}) \end{aligned} \tag{3.1}$$

.

In equation [3.1] all quantities refer to the upstream (incoming flow) state and u_x is measured in the computed (piston) frame. In the following discussion wherever it is necessary to distinguish upstream and downstream conditions they will receive subscripts $_1$ and $_2$ respectively. Of the remaining variables in equation [3.1] a is the gas sound (acoustic) speed, c_f is the fast mode wave speed; namely,

$$a = \sqrt{\frac{\gamma_g P_g}{\rho}}, \tag{3.2}$$

$$c_f^2 = \frac{1}{2}\{b^2 + a^2 + [(b^2 + a^2)^2 - 4b^2a^2\cos^2\theta_o]^{\frac{1}{2}}\}, \tag{3.3}$$

and θ_o is termed the "obliquity" of the shock. Note that the fast mode Mach number, M_f, in our simulations refers to the piston rather than the shock. The equivalent shock Mach number is

$$M_{fs} = \frac{|u_s|}{|u_x|} M_f. \tag{3.4}$$

The relationship between M_f and M_{fs} depends on the compression through the shock, which is generally a function of the solution itself and a function of time as P_{c2} changes. For most of the shocks described below, $M_f < M_{fs} \lesssim 4/3 M_f$. Our definition of β differs from that used frequently in the plasma physics literature and by WDV by a factor $\gamma_g/2$ = $5/6$. For all the simulations described, the upstream flow is characterized by $\rho = 1$, $u_x = -1$ and $N = 1/2$.

a) *Steady CR-MHD Shocks: Comparison with Previous Calculations*

We start with a comparison between properties of a steady state model computed by WDV and the approximately equivalent time asymptotic flow found by us. Previously

(Frank, Jones, & Ryu 1993) we showed good agreement between our results and the special "cold gas" analytic solutions examined by Webb. These comparisons are useful not only as tests of numerical methods, but also as demonstrations that the analytic steady states can be reached from some other initial state. In that context, however, we do not address in this paper issues associated with multiple steady state solutions found by WDV for CR-MHD shocks (see Donohue *et al.* 1993 for such a discussion regarding CR-gasdynamic shocks).

Figure 1 illustrates the evolution to an apparent dynamical steady post shock state of a simulated flow with upstream parameters $\beta = 1$, $\gamma_c = 4/3$, $\kappa = 10$, and $\theta_o = 60^o$. The resolution ratio was $n_r = 33$, so that the solution is well converged. The simulation was designed to match as closely as possible a solution presented by WDV in which $M_{fs} = 6.1$. (Note that their definition of c_f included P_c as a contributor to the sound speed. We have corrected for that in establishing an appropriate comparison.) The downstream partial pressures, P_{c2}, P_{g2} and P_{B2}, from WDV are indicated by dashed lines. The final u_s (and thus M_{fs}) is hard to predict exactly, because it depends on P_{c2}, P_{g2} and P_{B2} in nonlinear ways. In addition the WDV solution was read from a published figure (Figure 12 in DWVV), so we cannot match the two problems exactly. Nonetheless we find that all the partial pressures agree to within about 1% when normalized by the total momentum flux incident on the shock, $\bar{P}$, defined as

$$\bar{P} = \rho_1 u_s^2 + P_{B1} + P_{g1} + P_{c1}, \tag{3.5}$$

where to estimate u_s in the final steady shock, we assume the mass flux jump conditions are exactly satisfied across the shock. Then the required u_s in the frame of the upstream fluid can be written in terms of jump conditions across the shock in the computed frame as

$$u_s = \frac{[\rho u_x]}{[\rho]} - u_{x1} = \frac{r}{r\ -\ 1}|u_{x1}|, \tag{3.6}$$

where

$$r = \frac{\rho_2}{\rho_1}, \tag{3.7}$$

is the compression ratio through the full shock structure.

Figure 1 shows the flow at times $t/t_d = 325,\ 856$ and 1377. The shock transition has approached a steady state structure by $t/t_d = 856$. The rather large number of diffusion times needed for this shock to come into a dynamic equilibrium is consistent with previous gasdynamical CR shock calculations (*e.g.,* Jones & Kang 1990; Kang & Jones 1991). Jones & Kang (1990) pointed out that the rate for CR pressure to increase in response to diffusive acceleration depends not only on t_d, but also inversely on the CR adiabatic index. Physically that derives from the fact that pressure generated by non-relativistic particles scales as the particle momentum squared, while it increases only linearly with momentum for relativistic particles. Since $\gamma_c = 4/3$, the pressure in this model is assumed to come entirely from relativistic particles. The time to reach equilibrium also depends on the total change in P_c, of course, and so on N and M_{fs}.

As P_c builds within the shock precursor, a postshock density spike forms and then is left behind after the shock approaches equilibrium. The origin of this feature in strongly modified gasdynamic CR shocks was explained by Drury (1987) and independently by Jones & Kang (1990). Briefly, it reflects an adiabatic compression in the precursor at times before an initially strong subshock weakens in response to substantial adiabatic heating in the precursor. The tangential magnetic field participates in the development of that feature in oblique MHD shocks, as can be seen in Figure 1. These features are examples of time dependent effects that occur in the evolution of CR modified MHD shock structures and of which the steady analytical models can give no hint. We will discuss these time-dependent features in more detail in section 3e.

b) Steady CR-MHD Shocks: CR Acceleration Efficiency Maps

We now explore the influence of tangential magnetic fields on the CR acceleration efficiency. All previous related discussions have pointed out that strong, oblique fields tend to inhibit acceleration efficiency, at least in shocks of modest strength. However, the physical reasons for this have not been well determined. Based on their study of magnetosonic shocks Jun *et al.* (1993) made the reasonable conjecture that efficiency was reduced because a tangential field inhibits compression by stiffening the equation of state for the fluid . But the physics of oblique CR-MHD shocks is complex and nonlinear, so this question needs further examination to be understood. For this discussion we will define the CR acceleration efficiency to be the ratio of downstream CR pressure to upstream total momentum flux,

$$\epsilon_c = \frac{P_{c2}}{\bar{P}}. \tag{3.8}$$

In their study, WDV used analytical solutions to the steady CR-MHD shock equations for $M_f = 5$ and 1 and $\theta_o = 0^o,\ 30^o$ and 60^o to conclude that the obliquity of the field was the important acceleration inhibiting factor. To extend that effort we have completed a large set of simulations designed to map out time asymptotic values of ϵ_c as a function of the upstream parameters. A compact, visual way to examine how ϵ_c depends on the magnetic field properties in the flow is to construct maps of ϵ_c on a plane defined by the in-flowing normal and tangential Alfvénic Mach numbers,

$$M_x = \frac{u_x}{b_x} = \frac{u_x}{b\cos\theta_o} = \frac{M_g\beta^{\frac{1}{2}}}{\cos\theta_o} \propto \frac{1}{B_x}, \tag{3.8a}$$

and

$$M_z = \frac{u_x}{b_z} = \frac{u_x}{b\sin\theta_o} = \frac{M_g\beta^{\frac{1}{2}}}{\sin\theta_o} \propto \frac{1}{B_z}, \tag{3.8b}$$

where M_g is the sonic (gas) Mach number. For each map β is held constant. Shocks with a range of shock fast mode Mach numbers, M_{fs}, are obtained by varying M_x and M_z . For this purpose it is convenient to write M_f in terms of β and the total Alfvénic Mach number, M_b,

$$M_f^2 = \frac{2M_b^2}{\{1+\beta+[(1+\beta)^2 - 4\beta\cos^2\theta_o]^{\frac{1}{2}}\}}, \tag{3.9}$$

where

$$\frac{1}{M_b^2} = \frac{1}{M_x^2} + \frac{1}{M_z^2} = \frac{b^2}{u_x^2} = \frac{1}{\beta M_g^2}. \tag{3.10}$$

It is also useful to note that

$$M_b = M \sin 2\theta_o, \tag{3.11a}$$

$$\theta_o = tan^{-1}(M_x/M_z), \tag{3.11b}$$

and

$$M_z^2 = \frac{\rho u_x^2}{P_B} \xrightarrow[N=1/2]{} \frac{\bar{P}}{P_B} - 1 - 2\beta, \tag{3.11c}$$

where

$$M = (M_x^2 + M_z^2)^{\frac{1}{2}}. \tag{3.11d}$$

Figure 2 presents such time-asymptotic efficiency maps for $\beta = 20,\ 3,\ 0.8$, and 0.2. Each map was made from 25 simulations which uniformly covered the (M_x, M_z) plane to include $2 \leq M_{fs} \leq 16$. All of the simulations have $\gamma_c = 1.41$ and $n_r > 20$. The contours of CR acceleration efficiency, ϵ_c, are shown as solid lines and are labeled. The contours of fast mode shock Mach number, M_{fs}, are shown as dotted lines. They begin with the lowest contour set at $M_{fs} = 2$ in the lower left corner, increasing in steps of 2. Although the shocks in $\beta < 1$, $M_{fs} \sim 2$ flows would likely be influenced by finite "Alfvén transport" effects (*e.g.,* Jones 1993a), their inclusion here is useful in establishing trends within the CR-MHD solutions.

Some preliminary explanation should make the physics illustrated in Figure 2 easier to grasp. Note first that moving along an arc of constant radius (M) from the vertical axis corresponds to sweeping through θ_o from 0^o to 90^o. The total Alfvénic Mach number, M_b, and the fast mode Mach number, M_f, are maximum on this arc at $\theta_o = 45^o$. We note, as well, that for large M_z contours of M_f asymptote to lines $M_x = M_f$, while for large M_x these contours asymptote to lines $M_z = \sqrt{1+\beta}M_f$. Thus parallel and perpendicular shocks lie only at infinity in this plane.

It is easiest to begin by examining the $\beta = 20$ map, since MHD effects are minimal. There one can see that both the ϵ_c and M_{fs} contours are almost symmetric about $\theta_o = 45^o$ and that they are almost "parallel". There is a weak tendency at large obliquity for ϵ_c to decrease along a Mach number contour as θ_o increases. This is consistent with the conclusion reached by WDV. The effect is seen to be much stronger as one looks at successively smaller β; *i.e.,* as the field becomes dynamically more important. Consider, for example, the obliquity at which ϵ_c drops below 0.6 on the $M_{fs} = 8$ contour. As β decreases from 20 to 0.2 the value of the upstream obliquity at which these contours cross drops from $\theta_o \approx 63^o$ to $\theta_o \approx 47^o$. Thus, as the magnetic field becomes more dynamically significant shocks of constant M_{fs} and constant θ_o become less efficient CR accelerators found by others. Relation [3.11c] provides a clue to what is happening. Lines of constant M_z correspond to a constant ratio of in-flowing momentum flux to tangential magnetic pressure. When β is small $P_B/\bar{P} \propto M_z^{-2}$. For $\beta < 1$, ϵ_c contours in Figure 2 for high obliquity (quasi perpendicular) shocks are closely spaced and roughly parallel to the M_x

axis, and so parallel to constant $P_B/\bar{P}$. Thus, it is the dynamical capacity of magnetic pressure that seems to be most significant. This is reasonable and consistent with previous conclusions, of course. The effect is important only for small to moderate M_{fs}, since as M_{fs} becomes large M_z is restricted to values $> \sqrt{1+\beta}M_f \gg 1$. Then from equation [3.11c] $P_B/\bar{P}$ must be small for any β. This conclusion still does not discriminate among at least two possible physical reasons why the magnetic pressure plays this role. It could be, as suggested by Jun *et al.* (1993) just that the magnetic field reduces compression through the shock. Alternatively, it could reflect the need for the shock to do work on the field in compressing it, thus reducing the energy available to CR.

We will describe an experiment in the following subsection that may help to resolve that issue. But, first we should comment on another aspect of the efficiency contours in Figure 2. In this case we focus on quasi parallel shocks, so that M_x is relatively small. The efficiency of these shocks in the strong field ($\beta < 1$) regime seems to be higher at a given fast mode Mach number than when the field is weak. For instance, when $\theta_o = 20^o$ and $\beta = 0.2$, a $M_{fs} = 4$ shock leads to $\epsilon_c \approx 0.55$. At the same angle this efficiency requires $M_{fs} > 6$ when $\beta = 20$. On the face of it this seems to imply that the strong field has actually enhanced ϵ_c. But that would be a misinterpretation of the results. When $\beta \ll 1$ $M_f \approx M_b$; *i.e.,* the fast mode wave is much like an Alfvén wave for quasi parallel propagation and not very compressive. The large value of B that makes $c_f \approx b$ so large is mostly in the normal component. Using M_z as an indicator again, we can see that the dynamical impact of the tangential field cannot be very large in this regime. Thus, the fast mode Mach number underestimates the strength of the shock as it relates to diffusive acceleration. Indeed, examination of the structures of the CR-MHD shocks here reveals that the time asymptotic compression is large ($r > 4$) and comparable to that found in large β shocks having similar *acoustic* Mach number $M_g = M_b/\sqrt{\beta}$.

This last feature should emphasize that it would be wrong to expect that the existence of strong fields *necessarily* leads to a reduction in the efficiency of CR acceleration in astrophysical environments. That statement seems to apply only when the *tangential* magnetic pressure is strong. Thus, the orientation of the field is particularly important as $\beta < 1$.

c) *Steady Solutions: The Role of Compression and Magnetic Pressure*

Diffusive shock acceleration results from simultaneous compression and diffusion of CRs. Thus, the compression through the shock is itself obviously important to the efficiency of the process. But nonlinear effects complicate this simple statement. Even in CR-gasdynamical shocks the acceleration efficiency represents a balance in the competition between energy input into the CR and the gas. The incoming momentum flux does work on both the gas and the CRs, with the partitioning dependent on the required entropy production through the structure. In general a simple relationship between total compression and efficiency does not exist. When magnetic pressure is included the situation is even more complex, since compression of the magnetic field also requires work.

To explore the relative roles of the compression itself and the work done on the magnetic field in CR-MHD shocks, we have performed a simple set of numerical experiments. We carried simulations out to dynamical steady state postshock conditions for two values of the upstream obliquity, $\theta_o = 30^o$ and 60^o and two values of $\beta = 5,\ 1$. For each of these combinations we computed solutions for a range of CR adiabatic index, $4/3 \leq \gamma_c \leq 5/3$. By varying γ_c we change the compression through the shock, since $\gamma_c < 5/3$ softens the equation of state in a manner analogous to the way that the magnetic pressure stiffens it compared to the gas alone. All of the models used $M_f = 6$.

The results are illustrated in Figure 3. Figure 3a (upper left) shows the compression ratio, r, for the $\theta_o = 60^o$ models as a function of γ_c. As we expect from previous studies (*e.g.,* Drury & Völk 1981; Achterberg *et al.* 1984; Kang & Jones 1990) smaller γ_c leads to substantially greater compression through the shock. Note, however, that the compression is independent of β within our levels of uncertainty. As seen in Figure 3b (upper right) the efficiency, ϵ_c, does depend on γ_c, and thus on the compression. Note, however, that ϵ_c actually decreases with increasing compression; *i.e.,* as the mixed fluid develops a softer equation of state. That trend runs counter to the compression argument given by Jun *et al.* (1993) regarding the role of the magnetic field. But, ϵ_c *also* depends on β. More to the point, for a fixed compression (γ_c), ϵ_c is reduced as the tangential magnetic field becomes stronger. The change in ϵ_c can be accounted for roughly in the work done on the field itself. That can be seen in Figure 3d, which shows the normalized downstream magnetic pressure, $P_{B2}/\bar{P}$, for the same study. Two points are relevant. First, $P_{B2}/\bar{P}$ varies in a sense opposite to ϵ_c as γ_c varies. Second, the difference in $P_{B2}/\bar{P}$ between the weak and strong field cases is comparable to that in ϵ_c for the same two cases. Thus, crudely, at least, there has been an exchange between these two quantities. But, the nonlinear nature of these processes would lead us to expect a more complex situation quantitatively. Indeed, for the smaller obliquity case, $\theta_o = 30^o$, seen in Figure 3c, the efficiencies for different β are the same to within our $\sim 1\%$ uncertainties. The magnetic pressure, $P_{B2}/\bar{P}$ is smaller and has the same trend as in the other example. But, the low and high β values of $P_{B2}/\bar{P}$ differ by as much as 3%. Clearly, the other energy channels (gas pressure and kinetic energy) are also involved.

d) Time-dependent Effects: Nonisotropic Diffusion

From this point we will concentrate on evolutionary aspects of CR-MHD shocks as we can understand them through two-fluid models. For a variety of reasons it is likely that most astrophysically important CR shocks will not be of a steady character (see, *e.g.,* Jones 1992).

It is well known that the time asymptotic efficiency of two-fluid CR-gasdynamical shocks should be insensitive to the value or form of κ (Drury & Völk 1981; Jones 1993b). On the other hand the detailed properties of κ do influence the rate of shock evolution and the structure of the shock. We now briefly examine those issues for oblique CR-MHD shocks. Figure 4 presents properties of two simulations differing only in the properties of κ. The other properties of the flows were: $\beta = 1$, $M_f = 6$, $\gamma_c = 1.41$, $\theta_o = 60^o$.

Both shocks are shown at the same dynamical time, t, after the shock itself has reached an equilibrium structure. The dotted line represents a shock computed with constant $\kappa = 0.325$, while the solid line presents a flow with an anisotropic (obliquity dependent) κ with $\kappa_{\|} = 1.0 = 10 \times \kappa_{\perp}$, as defined in equation [2.10]. Upstream of the shock $\kappa = 0.325$ in this model also. Two points are obvious from the figure. First, the final post shock states, as represented by ρ and P_c, are the same in both simulations. This holds for the other state variables, as well. Second, the histories of the two shocks were different. One can trace the history by looking from left to right, since the shock moves away from the piston at the left boundary and to the first approximation the post shock fluid is at rest and not changing very fast (the next section discusses ways in which this is not quite true). The density spike identifies the approximate location of the shock when it reached dynamic equilibrium (see Figure 1). Thus, it is clear that this condition was achieved at an earlier time for the obliquity dependent diffusion model. That results from effective differences in diffusion times for the two models. For the constant diffusion model, $t_d = 0.325$, which matches the nominal (upstream) value for the obliquity dependent diffusion model, using the convention established in §2. But, through the precursor of that shock κ decreases from $\kappa = 0.325$ to $\kappa = 0.1$, so that effective values of t_d and x_d within the precursor and postshock flows are smaller as one proceeds downstream. That accounts in part for the sharper and higher post shock density spike seen in the variable diffusion model and the earlier achievement of a steady post shock state. Furthermore, the quicker evolution to a CR dominated post shock state with higher compression (due to the softer equation of state of the CR) explains why the position of the shock is not as far advanced for the obliquity dependent diffusion model. The shock speed decreases as the compression increases.

The more rapid shock evolution in the obliquity dependent diffusion model is related to the point emphasized by Jokipii (1987). He argues from test particle theory that with such diffusion models acceleration of individual particles will be quicker in highly oblique shocks, raising the possibility that particles of higher energy might be achieved in supernova remnant shocks than expected with isotropic, Bohm diffusion. Since we have not solved the diffusion-advection equation here we cannot comment directly on that issue except to make one point. More rapid acceleration also leads more quickly to the development of nonlinear feedback from the CR to the fluid. In highly oblique shocks in low β plasmas, that would mean a prompter onset of work done by CRs on the tangential magnetic field. The net result of that, of course, is a reduction in the total work done by the flow on the CR population. Since for $\gamma_c > 4/3$, P_c is dominated by low energy particles, these two behaviors are not necessarily contradictory. Clearly, a close look at this issue is warranted, however.

e) Time Dependent Effects: MHD Evolution

In Figure 5 we present the time development of a $M_f = 10$, strong field ($\beta = 1$), CR modified oblique ($\theta_o = 60^o$) MHD shocked flow. The evolution of the shock was calculated with $\gamma_c = 1.41$ and nonisotropic diffusion using $\kappa_{\|} = 1$ and $\kappa_{\perp} = 0.1$. Three time frames are shown at $t/t_d = 62, 123$, and 185 respectively. The plots demonstrate that the coupled CR-MHD evolution of the shock and especially the post shock flow are complex. Basic

qualitative evolution of the fluid and the CRs are similar to CR-gasdynamical shocks, however, and they have been discussed extensively in previously cited literature. We concentrate here on those features that are MHD in origin.

Through the shock the tangential magnetic field is enhanced. Mostly this is just compression, but in fast mode shocks there is an additional, Mach number dependent, enhancement through induction generated by shear in the shock transition (*e.g.,* Kantrowitz & Petschek 1966). That shear is evident in the u_z jump seen through the shock. The net tangential magnetic field change through any steady plane compression feature is given by the simple expression

$$r_B = \frac{B_{z2}}{B_{z1}} = r(\frac{M_x^2 - 1}{M_x^2 - r}), \tag{3.12}$$

where M_x refers to the condition upstream of the feature as measured in the rest frame of the feature. The factor on the right in brackets results from shear in the transition. The shock in Figure 5 corresponds to $M_x \approx 27$, so $r_B \approx r$. But for weaker shocks, including slow shocks the shear induced term is more significant. Equation [3.12] also shows the well-known difference between fast mode and slow mode waves. Whereas the sign of the change in tangential field and density are the same for fast waves ($M_x^2 \geq r > 1$), they are opposite for slow waves ($M_x^2 \leq 1$). This difference results from a reversal in the direction of the shear, $[u_z]$, relative to the normal velocity jump, $[u_x]$ through the feature (Kantrowitz & Petschek 1966). Then the induction (see the magnetic terms in equation [2.1]) also changes sign. This information helps sort out the evolution of the postshock flows seen in strong CR-MHD oblique shocks such as that shown in Figure 5. In this regard it is also useful to identify from the MHD dispersion relation that through a $\beta > 1$ fast mode compression, $(M_x\ [u_z]\ )/(M_z\ [u_x]\ ) > 0$. The opposite inequality applies for slow waves. For the flow described here $M_x/M_z > 0$, so the normal velocity gradient and the shear velocity gradient will have opposite signs in a fast compression, but the same sign in a slow compression.

Although the shock itself in the simulations shown reaches a dynamical steady state that is not true for the post shock density (and magnetic field) spike. Such spikes result from initially strong subshocks as they adjust to new CR dominated equilibrium conditions. At the start this feature is a fast mode feature, since the tangential field and gas density increase together with similar form at early times. It develops, after all, from the fast mode shock. At early times one can see that the shear velocity component, u_z that leads to inductive generation of field is negative behind the (fast mode) shock as expected from the relation just cited. But after the shock has reached an equilibrium, so that the density spike is left behind, a clear maximum in u_z is present just to the left of the peak in the density spike. That signals the formation of a slow wave from the density peak. This interpretation comes from the fact that $\partial u_x/\partial x <$ across almost the entire density spike, including the peak. Thus it is being slowly compressed. Hence the negative gradient in u_z must represent a slow mode compression. This development is even more obvious by the third and last time shown in Figure 5. By this time, in fact, the (right facing) slow mode compression has substantially reduced B_z on its downstream side, so that the density and magnetic spikes are clearly distinct.

Subsequently, the consequences of the slow wave are striking, in fact, as seen in Figure 6. This figure shows at later times the evolution of the density spike. One sees that the magnetic spike is eventually confined to the forward (right) edge of the density spike. In fact the forward edge of the density spike has steepened into a slow mode shock. This is demonstrated by the sharp jump in u_x at this point. The development of shocks in the transient density spike has not happened in any of the CR-gasdynamical shocks we have studied, so it seems to be strictly the result of the CR-MHD nature of this calculation. Although the density spike results from artificial nonequilibrium initial conditions, it may still be of much more than academic interest. Many astrophysical shocks will likely have transient properties. We have seen such density spikes in our simulations of SNR (Jones & Kang 1992) and in 2D simulations of supersonic dense clumps (Jones & Kang 1993; Jones, Kang & Tregillis 1993). The velocity jump at the slow mode shock in the spike is moderately large. It actually does lead to a small amount of CR particle acceleration downstream of the main shock. That can be seen through the small "bump" in the P_c curve at the last time shown in Figure 6. In addition, the shear and associated tangential current in the postshock flow resembles in some ways those inside the main shock. There they are accompanied by drift acceleration. For calculations in a planar geometry this does not appear, because the net energy flux vanishes (Jokipii 1982; Webb, Axford & Terasawa 1983). However, once that symmetry is broken, the drifts could play important roles just as they apparently due in perturbed oblique CR-MHD shocks (Decker 1988; Zank *et al.* 1990).

4. **Summary and Conclusions**

Utilizing a new numerical method built on the two-fluid model for diffusive shock acceleration we have conducted a study of CR modified oblique MHD plane shocks. The results of our numerical experiments are as follows:

1. For a fixed fast mode Mach number the particle acceleration efficiency, ϵ_c, of time asymptotic CR modified quasi perpendicular MHD shocks decreases as the upstream obliquity, θ_o, of the magnetic field increases. The dependence of ϵ_c on θ_o is greater for tangential fields whose amplification through the shock transition absorbs a significant portion of the incident momentum flux of the CR-MHD fluid. This limits the impact of the field on acceleration efficiency to moderately strong, quasi perpendicular shocks in strong field (low β) plasmas. These results confirm early analytical models and recent numerical models of perpendicular, magnetosonic shocks. Our experiments, however, map the relationships between ϵ_c and the MHD and CR properties more fully than previous studies and demonstrate that acceleration inhibition probably results from work done on the magnetic field rather than changes in fluid compression brought about by the presence of the field, for example.

2 For quasi parallel shocks, our results show that ϵ_c is higher at a given fast mode Mach number when $\beta < 1$ than when $\beta > 1$. While this would appear to suggest that strong fields enhance acceleration efficiency in quasi parallel shocks it actually results because in strong fields the quasi parallel fast wave speed is the Alfvén speed. The larger gas

sonic Mach number is a better measure of the strength and therefore the compression of $\beta < 1$ quasi parallel shocks. Thus strong fields do not necessarily imply a reduction in ϵ_c. It is the tangential magnetic pressure that is important.

3. The possibility of an anisotropic form for the CR diffusion coefficient, κ, does not affect the time asymptotic post shock state achieved by the CR modified MHD shocks. However, the history of CR-MHD shocks can be considerably modified by different forms of κ. In particular we have examined the effect of an obliquity dependent diffusion coefficient $\kappa(\theta)$ on CR-MHD shock evolution. In these models the compression of the tangential field through the shock reduces κ in the downstream regions and reduces the time necessary to achieve a steady state. That is consistent with earlier suggestions that anisotropic diffusion in quasi perpendicular shocks might allow individual cosmic ray particles to reach higher energies in astrophysical shocks having finite life times. The fact that strong field, quasi perpendicular shocks may at the same time become less efficient accelerators as the result of nonlinear effects needs to be studied using more complete models of diffusive acceleration. The two outcomes need not be contradictory, however, since the efficiency measures total energy exchange rather than the distribution of energies.

4. Time dependent CR-MHD effects may become significant downstream of the shock, even once the shock itself has approached a dynamical equilibrium. In particular if CR modification of the shock is sufficient to generate a postshock density spike, MHD influences in that spike can cause substantial evolution in the spike. We found that a strong post shock shear develops in response to magnetic stresses in strong, low β oblique shocks. In more realistic flows where planar symmetry constraints are broken this shear layer may yield additional CR acceleration through the production of drift currents. Further, a slow mode shock can form on the near edge of the post shock spike, possibly enhancing particle acceleration from the full structure.

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References

Achterberg, A. 1982, A& A, 98, 195.

Achterberg, A., Blandford, R.D, & Periwal, V. 1984, A&A, 98, 195.

Baring, M. G., Ellison, D. C., & Jones, F. C. 1993, ApJ, in press.

Berezhko, E. G. & Krymskii, G. F. 1988, Soviet Phys.-Uspekhi, 31, 27.

Bell, A. R. 1978, MNRAS, 182, 147.

Blandford, R.D, & Eichler, D. 1987, Phys Rept, 154, 1.

Brio, M. & Wu, C. C. 1988, J. Comp. Phys., 75, 400.

Chalov, S. V. 1988, Sov. Astron. Lett., 14, 114.

Decker, R. B. 1988, ApJ, 324, 566.

Donohue, D. J., Zank, G. P. & Webb, G. M. 1993, ApJ, in press.

Drury, L. O'C. 1983, Rep. Prog. Phys., 46, 973.

Drury. L. O'C. 1987, in *Proc. Sixth Int. Solar Wind Conf.*, ed. U. J. Pizzo, T. Holzer & D. G. Sime (NCAR/TN-360+Proc:), p.521.

Drury, L. O'C., & Völk, H. J. 1981, ApJ, 248, 344.

Drury, L. O'C., & Falle, S. A. E. G. 1986, MNRAS, 223, 353.

Duffy, P. 1992, A& A, 262, 281.

Duffy, P., Drury, L.O'C. & Völk, H. J. 1993, 23rd ICRC, Calgary, vol2, 259.

Ellison, D. C. & Eichler D. 1984, ApJ, 286, 691.

Falle, S. A. E. G., & Giddings J. R. 1987, MNRAS, 225, 399.

Frank, A., Jones, T. W. & Ryu, D. 1993, ApJS, Dec, ???.

Harten, A. 1983, J. Comp. Phys., 49, 357.

Jeffrey, A. 1966, *Magnetohydrodynamics* (London: Oliver and Boyd).

Jokipii, J. R. 1983, ApJ, 255, 716.

Jokipii, J. R. 1987, ApJ, 313, 846.

Jones, F. C. & Ellison, D. C. 1991, Space Sci. Rev., 58, 259.

Jones, T. W. 1992, in *Particle Acceleration in Cosmic Plasmas*, ed. G. P. Zank & T. K. Gaisser (American Institute of Physics: New York), p.148.

Jones, T. W. 1993a, ApJ, 413, 619.

Jones, T. W. 1993b, ApJS, Dec, ???.

Jones, T. W. & Kang, H. 1990, ApJ, 363, 499.

Jones, T. W. & Kang, H. 1992, ApJ, 396, 575.

Jones, T. W. & Kang, H. 1993, ApJ, 402, 560.

Jones, T. W., Kang H. & Tregillis, I. L. 1993, ApJ, Submitted.

Jun, B., Clarke, D. A. & Norman, M. L. 1993, Preprint, .

Kang, H. 1993, J. Korea Astron. Soc., 26, 1.

Kang, H., & Jones, T. W. 1990, ApJ, 353, 149.

Kang, H., & Jones, T. W. 1991, MNRAS, 249, 439.

Kang, H., Jones, T. W., & Ryu, D. 1992, ApJ, 385, 193.

Kantrowitz, A. & Petschek, H. E. 1966, in *Plasma Physics in Theory and Application*, ed. W. B. Kunkel (McGraw Hill: New York), p.148.

Kennel, C. F., Edmiston, J. P. & Hada, T. 1985, in *Collisionless Shocks in the Heliosphere: A Tutorial Review*, ed. R. G. Stone & B. T. Tsurutani (Amer. Geophys. Union: Washington, D. C.), p.1.

Kirk, J. G. & Heavens, A. F. 1989, MNRAS, 239, 995.

Ostrowski, M. 1988, MNRAS, 233, 257.

Roe, P. L. 1981, J. Comp. Phys., 43, 357.

Ryu, D., & Jones, T. W. 1993, ApJ, to be submitted.

Ryu, D., Kang, H. & Jones, T. W. 1993, ApJ, 405, 199.

Skilling, J. 1975, MNRAS, 172, 557.

Slavin, J. D., & Cox, D. P. 1992, ApJ, 392, 131.

Völk, H. J., Drury, L. O'C. & McKenzie, J. F. 1984, A& A, 130, 19.

Webb, G. M. 1983, A&A, 127, 97.

Webb, G. M., Axford, W. I. & Terasawa, T. 1983, ApJ, 270, 537.

Webb, G. M., Drury, L. O'C., Völk, H. J. 1986, A& A, 160, 335.

Zank, G.P., Axford, W. I., & McKenzie, J. F. 1990, A&A, 233, 275.

Appendix: Finite Difference Algorithms

To solve the MHD equations we have employed the Total Variation Diminishing (TVD) method introduced by Harten (1983). The TVD method solves the vector equation [2.1] without the source vector $\boldsymbol{S}$ in conservative, finite difference form:

$$(\boldsymbol{q}_i^{n+1})^* = \boldsymbol{q}_i^n + \frac{\Delta t}{\Delta x}(\bar{\boldsymbol{f}}^n_{i+\frac{1}{2}}(\tilde{\boldsymbol{q}}) - \bar{\boldsymbol{f}}^n_{i-\frac{1}{2}}(\tilde{\boldsymbol{q}})). \tag{A.1}$$

As usual, i is the spatial zone index; integers correspond to zone averages (centered values). The n index refers to time step. $\bar{\boldsymbol{f}}^n_{i\pm\frac{1}{2}}(\tilde{\boldsymbol{q}})$ is a 4 point numerical flux function that depends on the vector $\boldsymbol{q}$ and the original flux $\boldsymbol{F}(\boldsymbol{q})$. The TVD prescription of the numerical flux function $\bar{\boldsymbol{f}}^n_{i\pm\frac{1}{2}}(\tilde{\boldsymbol{q}})$ utilizes an approximate MHD Riemann solver (Roe 1981; Harten 1983; Brio & Wu 1988; Ryu & Jones 1993) to find estimates of the state variables at the zone boundaries. The method is an explicit, second order, Eulerian finite difference scheme. The TVD scheme also applies minimal numerical viscosity to the computed solution resulting in strong shocks that are represented with only 2 to 3 zones. Contact and rotational discontinuities are generally contained within $\lesssim$10 zones. For detailed descriptions and tests for the TVD method applied to the MHD problem see Ryu & Jones (1993).

CR dynamical feedback on the conducting fluid is handled in a two step process designed to preserve second order accuracy. Gradients in P_c accelerate and do work on the gas during a time step, Δt. To reflect that in the fluxes, $\bar{\boldsymbol{f}}^n_{i\pm\frac{1}{2}}(\tilde{\boldsymbol{q}})$, and thus improve the performance of the MHD Riemann solver, input Riemann solver momentum and energy densities are corrected to time centered values by a simple estimate of the force and work produced by the gradient in P_c,

$$\tilde{\boldsymbol{q}} = \boldsymbol{q}_i^n + \frac{\Delta t}{2}\boldsymbol{S}_i^n. \tag{A.2}$$

Then, after the TVD step represented in equation [A.1] with $\tilde{\boldsymbol{q}}$, the state vector $\boldsymbol{q}$ is updated for the effects of the source terms, $\boldsymbol{S}$; namely,

$$\boldsymbol{q}_i^{n+1} = (\boldsymbol{q}_i^{n+1})^* + \Delta t \boldsymbol{S}_i^n, \tag{A.3}$$

where $\boldsymbol{q}^*$ represents the solution to equation [A.1].

The CR energy is then updated through a two step procedure, which is also second order accurate. In the first step we solve the Lagrangian version of equation [2.5],

$$\frac{dE_c}{dt} = \frac{\partial E_c}{\partial t} + u\frac{\partial E_c}{\partial x} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial E_c}{\partial x}\right) - \gamma_c E_c \frac{\partial u}{\partial x} + S_e, \tag{A.4}$$

exactly as we do in the PPM two-fluid code, using a Crank-Nicholson scheme explained in Jones & Kang (1990). This is followed by a "remap" step,

$$E_{c,i}^{n+1} = (E_{c,i}^{n+1})^* - u_i^n\frac{\Delta t}{\Delta x}(g^n_{i+\frac{1}{2}}(E_c^*) - g^n_{i-\frac{1}{2}}(E_c^*)), \tag{A.5}$$

where

$$g^n_{i\pm\frac{1}{2}}(E^*_c) = (E^n_{c,i})^* \pm \frac{\Delta x}{2}(E^n_{c,i})^{*'}(1 \pm \frac{u^n_i \Delta t}{\Delta x}), \tag{A.6}$$

E^*_c is the solution to equation [A.5] and $(E_c)^{*'}$ is the monotone slope of E^*_c, (Van Leer 1974). In equations [A.4 - A.6] the x subscript has been dropped from u_x to simplify notation. This operator split method of solving equation [2.5] proved to offer the best stability and accuracy among several approaches we tried in numerical experiments during code development.

Figure Captions

Fig. 1.— Comparison between the analytical post-shock state (dashed line) and the results of a time-dependent simulation with approximately the same upstream conditions. Both models have $M_{fs} = 6.1$, $\theta_o = 60^o$, $\gamma_c = 4/3$ and $\beta = 1.0$. The time-dependent simulation used constant $\kappa = 10$. Shown are the density, gas pressure, magnetic pressure, normal velocity, tangential velocity and CR pressure. The simulation is shown at $t/t_d = 325, 856$, and 1377, where the diffusion time, t_d is defined in the text.

Fig. 2.— Maps of CR acceleration efficiency, ϵ_c, as a function of normal and tangential Alfvénic piston Mach number (M_x, M_z) for four different values of β. Solid lines are contours of ϵ_c, dashed lines are contours of fast mode shock Mach number, M_{fs}. ϵ_c contour levels are marked. In all the maps contours of M_{fs} are separated by steps of 2, with $M_{fs} = 2$ always the first contour seen in the lower left corner of map, and $M_{fs} = 16$ in the upper right. θ_o increase towards the x axis.

Fig. 3.— Results from a series of $M_{fs} = 6$ runs with $\beta = 5$ (solid line) and $\beta = 1$ (dashed line) for a range in the CR adiabatic index, γ_c. Upper Left: Compression ratio, $r = \rho_2/\rho_1$, as function of γ_c for upstream $\theta_o = 60^o$. $\beta = 5$ (solid) and $\beta = 1$ (dashed). Upper Right: CR acceleration efficiency ϵ_c as function of γ_c for upstream $\theta_o = 60^o$. Lower Left: CR acceleration efficiency ϵ_c as function of γ_c for upstream $\theta_o = 30^o$. Lower Right: Normalized downstream magnetic pressure, $\frac{P_{B2}}{P}$ as function of γ_c for upstream $\theta_o = 60^o$. Note that the $\beta = 1$ case is always more sensitive to γ_c and, therefore, to compression.

Fig. 4.— Comparison of two $\beta = 1$, $M_f = 6$, $\gamma_c = 1.41$ runs and different models of the CR diffusion coefficient. Dashed line shows a constant $\kappa = 0.325$ model. The solid line shows an obliquity dependent model (see text) Both represent the shock structures at $t/t_d = 431$.

Fig. 5.— The time evolution of a model with $M_f = 10$, $\theta_o = 60^o$, $\gamma_c = 1.41$, $P_{go} = 0.032$ and $\beta = 1.0$. κ is obliquity dependent as in Figure 4. Structure is shown at times $t/t_d = 62$, 123 and 185. Shown are the density, gas pressure, magnetic pressure, normal velocity, tangential velocity, tangential field, obliquity, diffusion coefficient and CR pressure.

Fig. 6.— Same as Figure 5 at times $t/t_d = 308, 615$ and 923. Note that here we have changed scale in x to focus on the evolution of the post shock density spike.